simulation of the Einstein-Podolsky-Rosen experiment in forth

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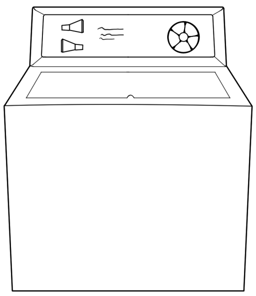
EuroForth 2021

V1.0

back to basics

```
: washer   wash  SPIN  rinse  SPIN  ;
```



Brodie, L. (1987), *Starting Forth*, Prentice-Hall.

- measuring the spin (magnetic moment) of a particle
- simulating spin measurements using forth: `epr-sim`
- quantum theory in a couple of slides
- factoring quantum states and entanglement
- exploring strong correlations in an entangled spin state using `epr-sim`
- EPR argument for incompleteness of QM [using entangled spins]
- exploring hidden variables explanations with `epr-sim`
- correlation coefficient and Bell's inequality for hidden variable theories
- computing Bell's inequality with `epr-sim`
- `epr-sim` design

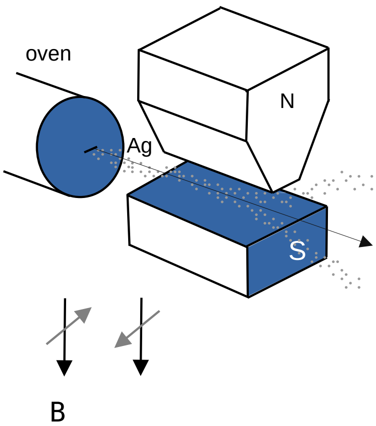
„Wäre es möglich, einen tüchtigen Physiker herbei [nach Frankfurt] zu ziehen, der sich mit dem Chemiker vereinigte und dasjenige heranbrächte, was so manches andere Kapitel der Physik, woran der Chemiker keine Ansprüche macht, enthält und andeutet; setzte man auch diesen in Stand, die zur Versinnlichung des Phänomens nötigen Instrumente anzuschaffen, so wäre in einer großen Stadt für wichtige, insgeheim immer genährte Bedürfnisse und mancher verderblichen Anwendung von Zeit und Kräften eine edlere Richtung gegeben.“

– *Johann Wolfgang Goethe,* 1814: Am Rhein, Main und Neckar.
In: Autobiographische Schriften. Band III, S. 297.

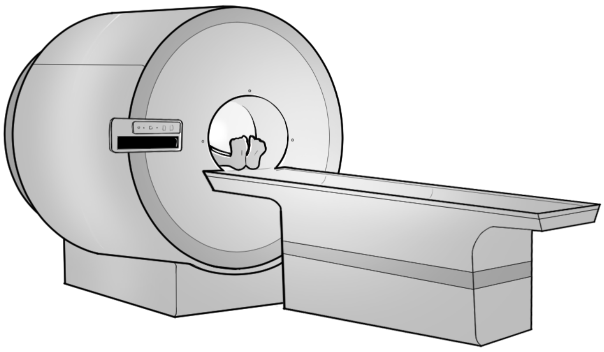
https://de.wikipedia.org/wiki/Physikalischer_Verein
https://www.goethe-university-frankfurt.de/63113635/Physics_of_yesterday

other kinds of spin machines

Stern-Gerlach experiment



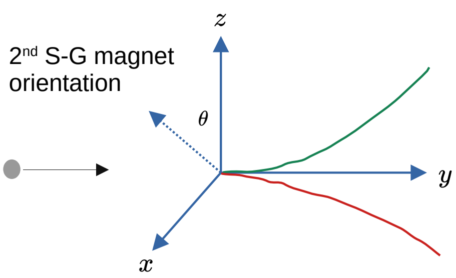
MRI scanner

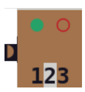


Artwork by Shreya

H. Schmidt-Böcking, et al., arXiv:1609.09311v1 [physics.hist-ph] 29 Sep 2016

single spin-1/2 particle in the “spin-up” quantum state





$P_u(0^\circ) = 1$

$P_u(60^\circ) = 3/4$

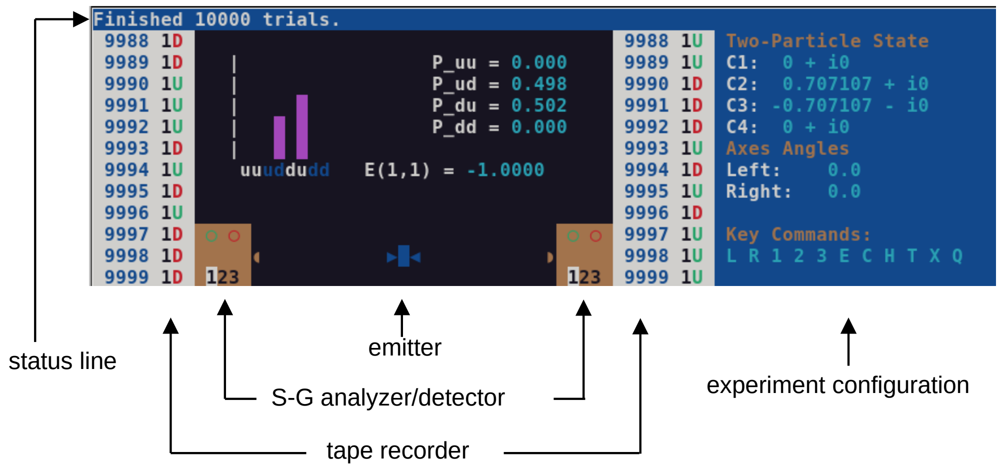
$P_u(120^\circ) = 1/4$

Simulation output from `epr-sim` for 0°, 60°, and 120°.

100 1U	100 2U	100 3D
101 1U	101 2D	101 3D
102 1U	102 2D	102 3U
103 1U	103 2D	103 3D
104 1U	104 2U	104 3D
105 1U	105 2U	105 3U
106 1U	106 2U	106 3D
107 1U	107 2U	107 3D
108 1U	108 2U	108 3D
109 1U	109 2U	109 3D
110 1U	110 2D	110 3D
111 1U	111 2U	111 3U

```
Q2p2s new dup
z1 z0 z0 z0 init-2p2s
EM set-qstate
0.0e 60.0e 120.0e rightDet map-angles
draw-experiment go
```

simulating spin measurements using forth: `epr-sim`[†]



† epr-sim.4th

quantum theory in a couple of slides

for a particle or system of particles in a defined *quantum state*, quantum theory

- predicts probabilities of possible measurement outcomes, *e.g.* $\{P_u, P_d\}$.
- *does not predict*, in general, results of individual measurements.

the above restrictions follow from the axioms and intepretation

- every possible measurement outcome of an *observable* has a *probability amplitude.*
- upon *measurement*, one of the possible outcomes is obtained, *e.g.* $\{+\hbar/2, -\hbar/2\}$.
- probability amplitudes follow a dynamics law (Schrödinger eqn.).
- some observables cannot have precise values simultaneously, *e.g.* $\{x, p_x\}$, $\{s_x, s_z\}$.

quantum states for computer scientists

the *quantum state* is a list of associations between measurement outcomes and probability amplitudes

```
( (mo1 c1) (mo2  c2) … (mo_n c_n) )
```

ex1: single spin-1/2 particle state observed along a specified axis

```
( (up c1) (down c2) )
```

ex2: two spin-1/2 particles state observed along a specified common axis

```
( ((upA upB) c1) ((upA downB) c2) ((downA upB) c3) ((downA downB) c4) )
```

c_i are complex numbers

require $|c_1|^2 + |c_2|^2 + \ldots = 1$

factoring two-particle quantum states

can we factor two-particle states as a product of separate one particle states?

```
(equal '( ((uA uB) c1) ((uA dB) c2) ((dA uB) c3) ((dA dB) c4) )

        (product '( (uA z1) (dA z2) )  '( (uB z3) (dB z4) ) ) )
```

for consistency with probability interpretation, `product` must use the relations

$$
\begin{aligned}
c_1 &= z_1 z_3 \;\rightarrow\; |c_1|^2 = |z_1|^2|z_3|^2 \\
c_2 &= z_1 z_4 \;\rightarrow\; |c_2|^2 = |z_1|^2|z_4|^2 \\
c_3 &= z_2 z_3 \;\rightarrow\; |c_3|^2 = |z_2|^2|z_3|^2 \\
c_4 &= z_2 z_4 \;\rightarrow\; |c_4|^2 = |z_2|^2|z_4|^2
\end{aligned}
$$

then, our Lisp expression evaluates to `T`.

two-particle states can be factored if measurement of one particle is independent of measurement of the other.

unfactorable two-particle quantum states

example of an unfactorable (*entangled*) state:

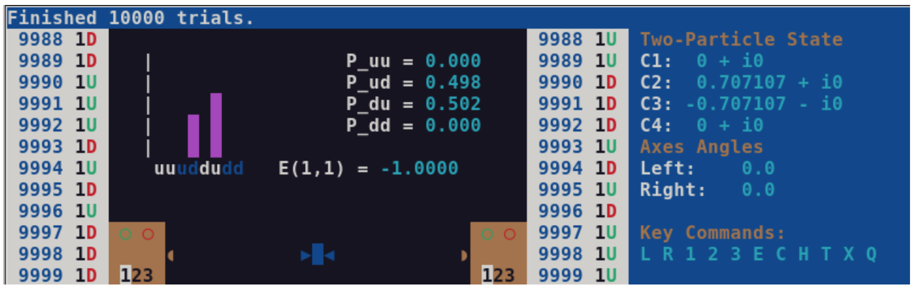
***singlet* two-particle spin state** $c_1 = 0,\ c_2 = 1/\sqrt{2},\ c_3 = -1/\sqrt{2},\ c_4 = 0$

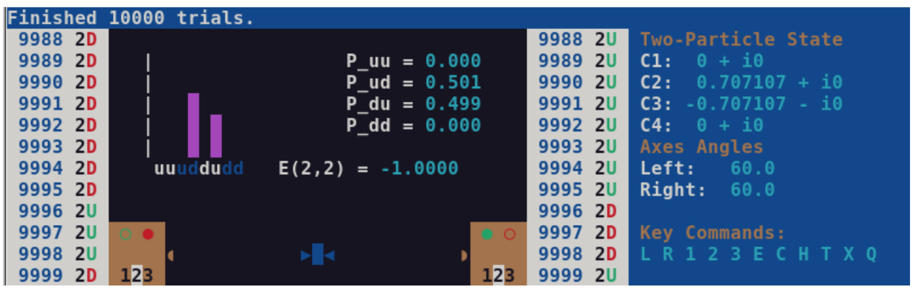
$$
\begin{aligned}
c_1 &= z_1 z_3 = 0 \\
c_2 &= z_1 z_4 = 1/\sqrt{2} \\
c_3 &= z_2 z_3 = -1/\sqrt{2} \\
c_4 &= z_2 z_4 = 0
\end{aligned}
$$

no assignment of z_1, z_2, z_3, z_4 can satisfy the above equations.

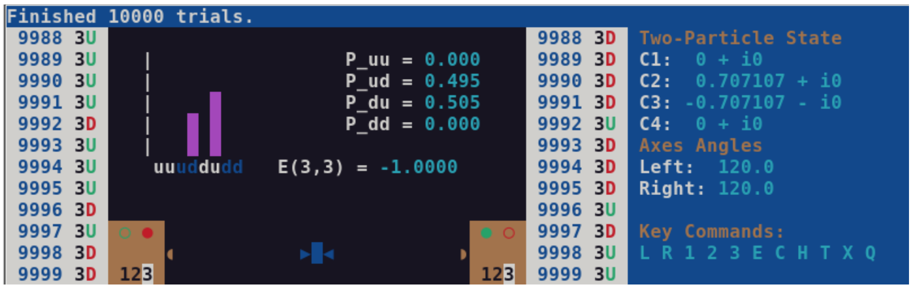
our Lisp expression evaluates to `NIL` for entangled states.

exploring strong correlations in an entangled state using `epr-sim`





magic of the singlet state



- each particle, (left and right-going) has equal chance (50%) of spin U or D with respect to any axis.
- measurements for both are perfectly *anti-correlated* when both detectors are set to the same angle – this is the case *for all angles*.

EPR argument for incompleteness of QM [using entangled spins]

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

- left and right detectors can be arbitrarily far apart, and at different distances from the source.
- after a measurement is made on the left, result of measurement on the right, along the same axis, *may be predicted with certainty*.
- measurement on the left cannot in any way disturb the measurement made on the right.
- The axis selection may be random, for example along z-axis (0°) or along x-axis (90°).

therefore, the result of spin measurement on the right exists independently of the measurement on the left, and the quantum state description is incomplete.

A. Einstein, B. Podolsky, and N. Rosen, Physical Review 47, 777 (1935).
D. Bohm and Y. Aharonov, Physical Review 108, 1070 (1957).

hidden variable explanations for spin correlations

assume there exists a *complete* state description with parameter(s) we don't know.

let λ be a random bit (0 or 1) generated at source, and state be specified by

```
λ      s>z zvalue λ1
λ  0=  s>z zvalue λ2
```

```
( ((u u) 0) ((u d) λ1) ((d u) λ2)) ((d d) 0) )
```

which is a factorable (unentangled) state.

λ = 0: `( ((d u) -1) )`

λ = 1: `( ((u d)  1) )`

outcomes are fully determined along 0° when *hidden variable* λ is known:

$A(\lambda{=}0,\ 0^\circ) = \mathrm{D}$, $A(\lambda{=}1,\ 0^\circ) = \mathrm{U}$, $B(\lambda{=}0,\ 0^\circ) = \mathrm{U}$, $B(\lambda{=}1,\ 0^\circ) = \mathrm{D}$

can we find deterministic laws which agree with QM statistics for singlet state?

possible assignments for spin measurements are shown in table

λ	$A(\lambda,\ \theta_i)$			$B(\lambda,\ \theta_j)$		
	1	2	3	1	2	3
0	D	D	D	U	U	U
0	D	D	U	U	U	D
0	D	U	D	U	D	U
0	D	U	U	U	D	D
1	U	D	D	D	U	U
1	U	D	U	D	U	D
1	U	U	D	D	D	U
1	U	U	U	D	D	D

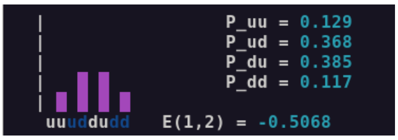
$i,\ j$	P_{uu}	P_{ud}	P_{du}	P_{dd}
1, 1	0	½	½	0
2, 2	0	½	½	0
3, 3	0	½	½	0

exploring hidden variables explanations with `epr-sim`

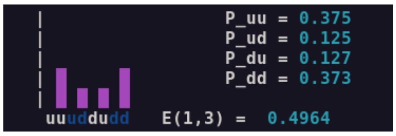
when detector settings are different, QM statistics *do not match* the table statistics.

i*, *j

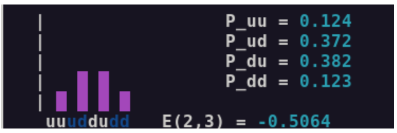
1, 2



1, 3



2, 3



λ	$A(\lambda,\theta_i)$			$B(\lambda,\theta_j)$		
	1	2	3	1	2	3
0	D	D	D	U	U	U
0	D	D	U	U	U	D
0	D	U	D	U	D	U
0	D	U	U	U	D	D
1	U	D	D	D	U	U
1	U	D	U	D	U	D
1	U	U	D	D	D	U
1	U	U	U	D	D	D

i, *j*	P_{uu}	P_{ud}	P_{du}	P_{dd}
1, 2	¼	¼	¼	¼
1, 3	¼	¼	¼	¼
2, 3	¼	¼	¼	¼

correlation coefficient and Bell's inequality for hidden variable theories

E is defined to be the average of the product of the two spin measurements, with U ≡ +1 and D ≡ −1.

$$E = P_{uu} + P_{dd} - P_{ud} - P_{du}$$

E is also the *correlation coefficient* (reflective correlation coefficient†).

E depends on the two detector angles, θ_L and θ_R (left and right).

J. S. Bell proved‡ that *any* local hidden variable theory must give Es satisfying the following inequality for the singlet state

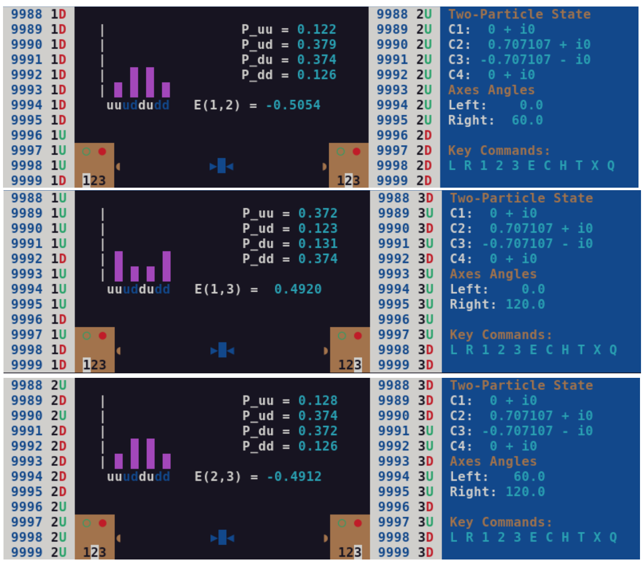
$$|E(1,2) - E(1,3)| - E(2,3) \leq 1$$

where (1,2), (1,3), and (2,3) correspond to left and right detector angle selector settings.

† https://en.wikipedia.org/wiki/Pearson_correlation_coefficient

‡ J. S. Bell, Physics 1, 195 – 200 (1964).

computing Bell's inequality with `epr-sim`




```
10000 value NTRIALS
Singlet EM set-qstate
: f3dup 2 fpick 2 fpick 2 fpick ;

: measure-along ( leftaxis rightaxis -- ) ( F: -- E )
    reset-counts
    rightDet set-axis  leftDet set-axis
    NTRIALS run-fixed-trials drop
    expectation-value ;

: measure-lhs ( -- ) ( F: deg1 deg2 deg3 -- lhs )
    f3dup
    leftDet  map-angles
    rightDet map-angles
    2 3 measure-along
    1 2 measure-along
    1 3 measure-along
    f- fabs fswap f- ;

0.0e 60.0e 120.0e measure-lhs
```

$$|E(1,2) - E(1,3)| - E(2,3) = 1.5 \not\leq 1$$

exercise in using `epr-sim`

Consider the two-particle spin state:

$$c_1 = ½ \quad c_2 = i(½) \quad c_3 = i(½) \quad c_4 = -½$$

Obtain the joint probabilities P_{uu}, P_{ud}, P_{du}, P_{dd} and the correlation, E, for the following pairs of axes:

$$\begin{aligned}
\mathbf{1,\ 1} &\coloneqq 0^\circ,\ 0^\circ \\
\mathbf{2,\ 2} &\coloneqq 60^\circ,\ 60^\circ \\
\mathbf{3,\ 3} &\coloneqq 120^\circ,\ 120^\circ \\
\mathbf{1,\ 2} &\coloneqq 0^\circ,\ 60^\circ \\
\mathbf{1,\ 3} &\coloneqq 0^\circ,\ 120^\circ \\
\mathbf{2,\ 3} &\coloneqq 60^\circ,\ 120^\circ
\end{aligned}$$

Setup commands:

```
0.0e 60.0e 120.0e  f3dup
leftDet  map-angles  rightDet map-angles
Q2p2s new constant TestState
z1/2 zdup i* zdup z1/2 znegate TestState init-2p2s
TestState EM set-qstate
draw-experiment go
```

Do the measurements appear to show any correlation for these settings?

Is the two-particle state *entangled*, or is it *factorable* into independent one particle states (Bell's inequality cannot be used for this state)?

`epr-sim` design: forth libraries

forth libraries

`mini-oof.x` compact, object-oriented programming word set by Bernd Paysan†
`ansi.x` ANSI terminal control library‡
`strings.x` simple strings library‡

`forth scientific library`‡
  `fsl-util.x`
  `complex.x` (**#60**)
  `ran4.x` (**#24**)

† Detailed Description of Mini-OOF
‡ kForth-64 forth source examples
††The Forth Scientific Library ; Forth-94 and Forth-2012 compliant Forths may also use kForth versions of FSL modules with the addition of a few compatibility definitions.

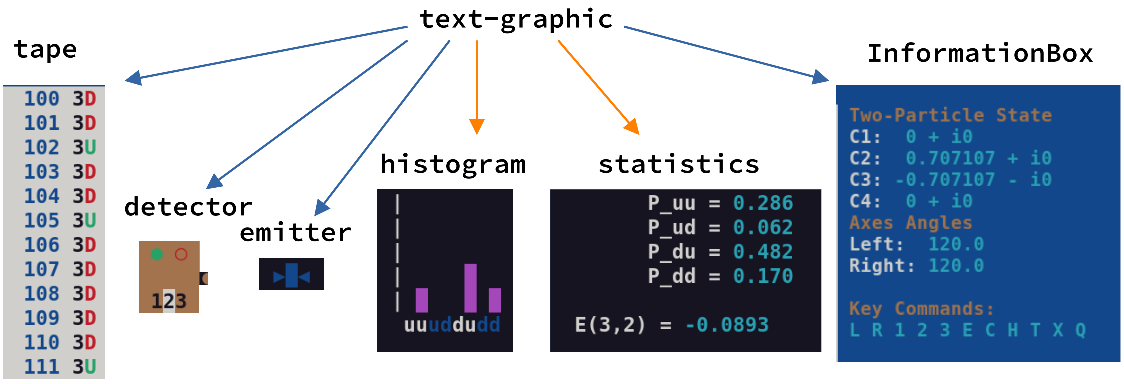
`epr-sim` design: two-particle spin-1/2 state

```
object class
    complex var C1  \ amplitude of |11> component
    complex var C2  \ amplitude of |10> component
    complex var C3  \    "          |01> component
    complex var C4  \    "          |00> component
    method init-2p2s ( o -- ) ( F: z1 z2 z3 z4 -- )
    method normalize ( o -- )
    method exchange  ( o -- )  \ exchange particle labels
    method P_up      ( o -- ) ( F: stheta ctheta -- P_up )
    method M_up      ( o -- ) ( F: stheta ctheta -- C1' C2' C3' C4' )
    method M_down    ( o -- ) ( F: stheta ctheta -- C1' C2' C3' C4' )
end-class Q2p2s  \ two-particle, bipartite quantum state
```

method `normalize` ensures total probability = 1
method `P_up` computes $P_{uu}(\theta_1)$ + $P_{ud}(\theta_1)$

epr-sim design: oop

virtual experiment components are derived from the `text-graphic` class



some visual elements inspired by N. D. Mermin, Physics Today, April 1985, pp 38 -- 47.

dedication

My presentation is dedicated to the memory of professors from whom I learned quantum theory,

Prof. Shi-Yu Wu

Prof. Eugen Merzbacher

appendix: product state of single particles

we have to map $c_i = f_i(z_1, z_2, z_3, z_4)$ with following constraints

$$P_{uu} + P_{ud} + P_{du} + P_{dd} = |c_1|^2 + |c_2|^2 + |c_3|^2 + |c_4|^2 = 1$$

$$P_u^A = P_{uu} + P_{ud} \rightarrow |z_1|^2 = |c_1|^2 + |c_2|^2$$

$$P_d^A = P_{du} + P_{dd} \rightarrow |z_2|^2 = |c_3|^2 + |c_4|^2$$

$$P_u^B = P_{uu} + P_{du} \rightarrow |z_3|^2 = |c_1|^2 + |c_3|^2$$

$$P_d^B = P_{ud} + P_{dd} \rightarrow |z_4|^2 = |c_2|^2 + |c_4|^2$$